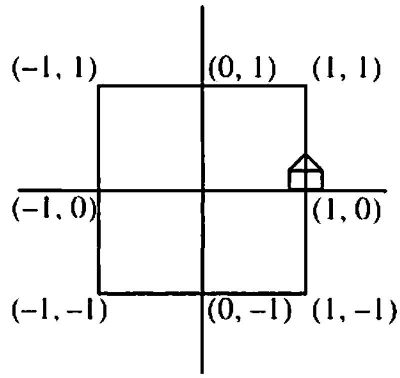
HON PRECALCULUS Name: Solutions

Francois and his Pedometer



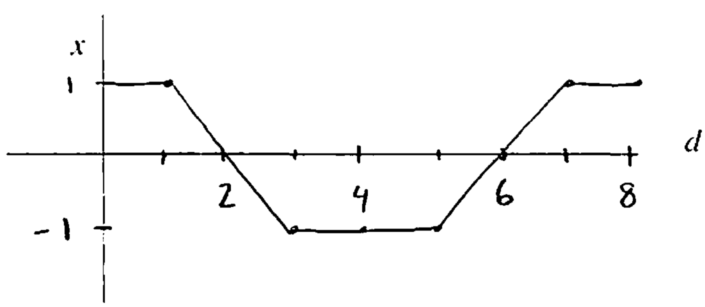
Instructions: Please show all work and/or justify all answers.

Francois lives in a house at (1, 0) Coordinate Plane Lane in Cartesian, Switzerland. He likes to go out walking frequently and always wears a pedometer so that he knows how far he has walked. He also always takes the same route around his neighborhood as pictured in the diagram. His route takes him 1 block north, 2 blocks west, 2 blocks south, 2 blocks east, and then 1 block north again, back to his house. Note that Francois' position can be given by an (x, y) pair. For example, after he has walked one block, he is at position (1, 1).

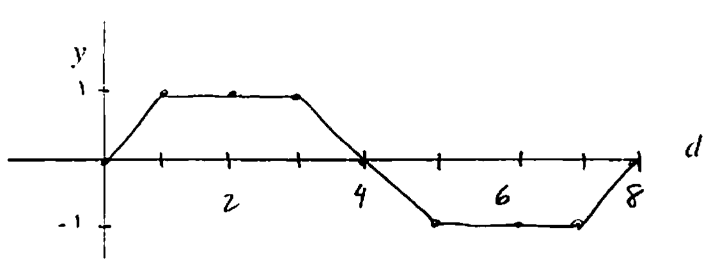
1. When Francois has walked 5 blocks, what is his coordinate position?

(−1, −1)

2. Suppose Francois walks 1 block in 1 minute. Create a graph of x vs. d, where x is the x-coordinate of his location and d is the distance Francois has walked. When $d = 0$, he is at his house.



3. Create a graph of y vs. d, where y is the y-coordinate of his location and d is the distance he's traveled.



4. Above, we thought of x and y as functions of d, the distance traveled. In fact, using function notation, we could have written $x(d)$ and $y(d)$ to emphasize this point. How do you know that these are actual mathematical functions?

They pass the vertical line test

5. The graph of x vs. d and y vs. d are very similar. Thinking of $y(d)$ as a transformation (horizontal shift) of $x(d)$, describe $y(d)$ in terms of $x(d)$. [This means write $y(d)$ = ________.] If you have difficulty, first describe the relationship with words and then write an algebraic statement.

$$y(d) = x(d-2)$$

6. Francois often walks his standard square route more than once.

a. When Francois has walked 10 blocks, what is his location?

$(0,1)$

b. To rephrase the question, when $d = 10$, what are $x(10)$ and $y(10)$?

$x(10) = 0 \quad y(10) = 1$

c. Determine each of the following.

i. $x(16)$	ii. $y(16)$	iii. $x(4.5)$	iv. $y(4.5)$	v. $y(77.5)$	vi. $x(1362)$
1	0	-1	$-\frac{1}{2}$	-1	0

7. On occasion, Francois likes to walk backwards around his route. Not only does he head south when he leaves his house, but he also walks backwards so that his pedometer starts registering negative values!!

a. Francois' pedometer says that $d = -53$. Determine where Francois is.

$(-1,1)$

b. Determine $x(-21.5)$ and $y(-21.5)$.

$x(-21.5) = -\frac{1}{2} \quad y(-21.5) = 1$

8. As you may know, a function is said to be periodic when it repeats. The period of a function is the length of the interval over which it is unique. Making copies of the function over this interval generates the entire function. Presumably you've noticed that these functions are periodic. What is the period of $x(d)$? What is the period of $y(d)$?

Period of each is 8.

9. One day, Francois finds himself at an x–coordinate of 0. He looks down to check his pedometer to see how far he has walked. Much to his dismay, he realizes that it is broken! Francois has no idea how far he has walked. Help Francois by figuring out the set of all distances that he could have possibly walked. (First, list possible distances, and then look for a way to generalize your list.)

$2 + 4k$, where k is an integer $(k \in \mathbb{Z})$

10. Tragedy strikes the next day when Francois' pedometer again fails to work. This time, he finds himself with a y–coordinate of 0. Determine the set of all distances that he could have possibly walked.

$4k$, where k is an integer $(k \in \mathbb{Z})$

11. a. If Francois had not yet made it once around the block, and he found himself with an x–coordinate of 1/2, what are the two possible distances he could have walked?

1.5 or 6.5

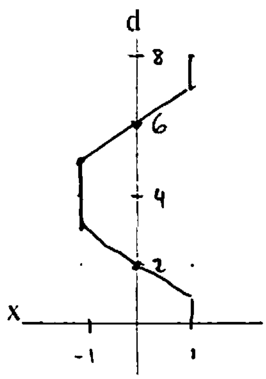
b. If he could have walked any number of times around the block, what is the set of all distances that he could have possibly walked?

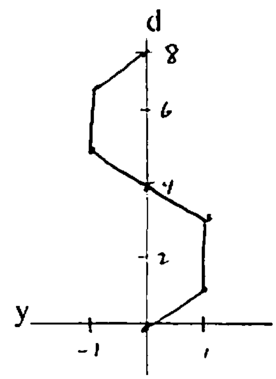
$1.5 + 8k$ or $6.5 + 8k$, $k \in \mathbb{Z}$

12. If Francois had found himself with a y–coordinate of $-1/4$, what is the set of all distances that he could have possibly walked?

$4.25 + 8k$ or $7.75 + 8k$, $k \in \mathbb{Z}$

13. Neatly sketch the <u>inverse</u> of both $x(d)$ and $y(d)$. Be sure to label axes appropriately.

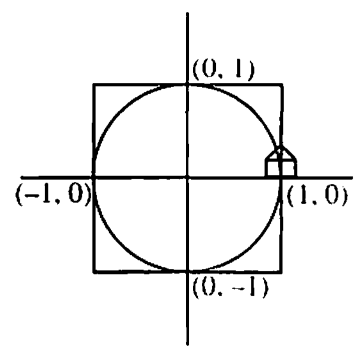




14. Is the inverse of $x(d)$ a function? Is the inverse of $y(d)$ a function? Justify your answers.

No; they both fail the vertical line test.

Francois has discovered a new walking route! He got sick of the square route because it seemed irrational so much of the time and found one that has a different mathematical feel. This path around his neighborhood happens to be a perfect circle. In walking this route, Francois has discovered many interesting things about his position based on the distance he's traveled. One thing he notices is that when he gets to the position (0, 1), his pedometer no longer registers two blocks! (Francois isn't literally walking in complete blocks anymore, but he can still measure his distance in terms of the number of blocks traveled.)



15. How many "blocks" does it register?

$\frac{\pi}{2}$

16. a. Assuming that this is his first trip around, how many "blocks" has Francois gone when he is at (–1, 0)?

π

b. How many "blocks" has he gone when he is at (0, –1)?

$\frac{3\pi}{2}$

c. When he's back at (1, 0)?

2π

17. We can still think of Francois' x and y coordinate as functions of the distance that he's traveled on the path. That is, we can speak of $x(\pi/2)$ as referring to the x–value of the coordinate Francois is at after travelling $\pi/2$ blocks. (Yes, his pedometer has a setting so that it will give the number of blocks traveled in terms of π. Honest!)

a. Where is Francois after walking $\pi/4$ blocks? That is, what is $x(\pi/4)$ and what is $y(\pi/4)$? (<u>Hint</u>: a 45-45-90 triangle will help!) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

b. What is $x(3\pi/4)$? $\frac{-\sqrt{2}}{2}$

c. What is $y(3\pi/4)$? $\frac{\sqrt{2}}{2}$

d. What is $y(5\pi/4)$? $\frac{-\sqrt{2}}{2}$

e. What is $x(7\pi/4)$? $\frac{\sqrt{2}}{2}$

18. Francois needs some help again! Though he knows how far he's traveled when he's at a position such as (0, 1) or ($\sqrt{2}/2, \sqrt{2}/2$), he finds himself at a variety of other coordinates.

a. Assuming that he is on his first trip around the route, how far has he traveled when he finds himself at ($\sqrt{3}/2$, 1/2)? (<u>Hint</u>: a 30-60-90 triangle will help!)

$\frac{\pi}{6}$

b. How would you change your answer if you didn't assume that this was his first trip? In other words, how far *could* he have traveled to arrive at ($\sqrt{3}/2$, 1/2)?

$\frac{\pi}{6} + 2\pi k$

c. On his first lap, how far has Francois traveled when he finds himself at ($-\sqrt{3}/2$, 1/2)? On his second lap?

$\frac{5\pi}{6}$; $\frac{17\pi}{6}$

d. On his first lap, how far has Francois traveled when he finds himself at ($-1/2, -\sqrt{3}/2$)? On his third lap?

$\frac{7\pi}{6}$; $\frac{31\pi}{6}$

19. Use an equation to describe the set of all (x, y) pairs that will be Francois' coordinates.

$$x^2+y^2=1$$

20. Francois has just left his house and headed out on his regular circular route. He notes that his x–coordinate is $\sqrt{2}/5$. Can you determine his y–coordinate? If so, do so. If not, explain why not.

No – it could be either $\frac{\sqrt{23}}{5}$ or $\frac{-\sqrt{23}}{5}$.

21. A friend tells Francois that most calculators have functions that can evaluate his x or y position for any given number of blocks he has traveled. What are those functions, and how could they be used to help Francois figure out one possible distance he could have traveled if he has a position of ($\sqrt{3}/2$, 1/2)?

In <u>radian</u> mode: $x(d) = \cos d$ $\quad y(d) = \sin d$

Either use $\cos d = \frac{\sqrt{3}}{2} \rightarrow d = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin d = \frac{1}{2} \rightarrow d = \sin^{-1}\left(\frac{1}{2}\right)$

22. Explain how a calculator could help Francois figure out the set of all possible distances traveled if he finds himself at an x–coordinate of –0.75. Find those sets of distances.

Since $\cos d = -0.75$, then $d = \cos^{-1}(-0.75) \approx 2.42$.

All possible distances are $2.42 + 2\pi k$ or $2\pi - 2.42 + 2\pi k$ where $k \in \mathbb{Z}$.